Math 2A Quiz 1 Version 1
Fri Jan 15, 2016

NAME YOU ASKED TO BE CALLED IN CLASS: ______________________

SCORE: $13\frac{1}{2}$ **/ 30 POINTS**

1. NO CALCULATORS OR NOTES ALLOWED
2. UNLESS STATED OTHERWISE, YOU MUST SIMPLIFY ALL ANSWERS
3. SHOW PROPER CALCULUS LEVEL WORK TO JUSTIFY YOUR ANSWERS

Consider the IVP $y' = 2y^2 - 3x,\quad y(1) = -2$. Use Euler's method with $h = 0.2$ to estimate $y(1.4)$. **SCORE:** 0 **/ 4 PTS**

$y(x+h) = y(x) + y'(x)(h)$

$y(1.2) = y(1) + [2(1)^2 - 3(1)](0.2)$

$= -2 - 0.2 = -2.2$

$y(1.4) = y(1.2) + [2(1.2)^2 - 3(1.2)](0.2)$

$= -2.2 + (2.88 - 3.6)(0.2)$

$= -2.2 - 0.144$

$= -2.344$

$$\begin{array}{r} -3.60 \\ +2.88 \\ \hline -0.72 \\ 0.2 \\ \hline -0.144 \\ -2.2 \\ \hline -2.344 \end{array}$$

Determine if $y = A\sqrt{x} + \dfrac{B}{x^2} + \dfrac{x^2}{4}$ is a family of solutions of the DE $4x^2y'' + 10xy' - 4y = 5x^2$. **SCORE:** 6 **/ 6 PTS**

State your conclusion clearly.

$\longrightarrow y = Ax^{\frac{1}{2}} + Bx^{-2} + \frac{1}{4}x^2 = x^2\left(Ax^{-\frac{3}{2}} + Bx^{-4} + \frac{1}{4}\right)$

$y' = \frac{1}{2}Ax^{-\frac{1}{2}} + (-2Bx^{-3}) + \frac{1}{2}x$

$= \underbrace{\frac{1}{2}Ax^{-\frac{1}{2}} - 2Bx^{-3} + \frac{1}{2}x = x\left(\frac{1}{2}Ax^{-\frac{3}{2}} - 2Bx^{-4} + \frac{1}{2}\right)}$ 1

$y'' = \underbrace{-\frac{1}{4}Ax^{-\frac{3}{2}} + 6Bx^{-4} + \frac{1}{2}}$ 1

$4x^2y'' + 10xy' - 4y = 4x^2\left(-\frac{1}{4}Ax^{-\frac{3}{2}} + 6Bx^{-4} + \frac{1}{2}\right)$

$+ 10x^2\left(\frac{1}{2}Ax^{-\frac{3}{2}} - 2Bx^{-4} + \frac{1}{2}\right)$

$- 4x^2\left(Ax^{-\frac{3}{2}} + Bx^{-4} + \frac{1}{4}\right)$ 1

$= (x^2)\left[(-1+5-4)(Ax^{-\frac{3}{2}}) + (24-20-4)(Bx^{-4}) + (2+5-1)\right]$

$= \underbrace{6x^2} \neq 5x^2 \quad \therefore\ y = A\sqrt{x} + \frac{B}{x^2} + \frac{x^2}{4}$ is $\underbrace{\text{not}}$ 1

2 a family of solutions of the DE.

$P < P_s \Rightarrow$ extinct.

In certain population models, a group will go extinct if and only if its population is below a certain level (called the survival threshold P_S). Write a differential equation for the population of a group which is going extinct, if the rate of change of its population is proportional to the difference between the threshold and the existing population. **Justify your answer properly, but briefly.**
NOTE: The signs of all constants should be stated clearly. Let P be existing population

SCORE: $3\frac{1}{2}$ / 4 PTS

$$\frac{dP}{dt} = k(P_s - P)$$

if $P_s - P < 0 \Rightarrow P > P_s$ $\quad \frac{dP}{dt} > 0$, $\therefore k < 0$

if $P_s - P > 0 \Rightarrow P_s > P$ $\frac{1}{2}$

$\frac{1}{2}$ $\frac{dP}{dt} < 0$, $\therefore k < 0$

$$\therefore \frac{dP}{dt} = k(P - P_s)$$ $2\frac{1}{2}$

What does the Existence and Uniqueness Theorem tell you about possible solutions to the IVP $\frac{dy}{dx} = \frac{\sqrt[3]{y-2}}{x+6}$, $y(8) = 2$? **Justify your answer properly, but briefly.**

SCORE: 4 / 4 PTS

around point $(8, 2)$

$$f(x,y) = \frac{\sqrt[3]{y-2}}{x+6} = \frac{(y-2)^{\frac{1}{3}}}{x+6}$$

$$f_y = \frac{1}{3(x+6)}(y-2)^{-\frac{2}{3}}$$

$$= \frac{1}{3(x+6)} \cdot \frac{1}{\sqrt[3]{(y-2)^2}}$$ 1

$f(x,y)$ is continuous at $(8,2)$ ✓

f_y is not continuous at $(8,2)$. X $1\frac{1}{2}$

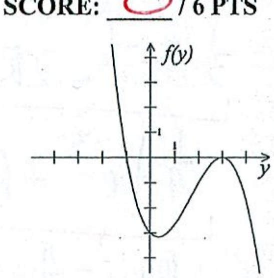
$\therefore \sqrt[3]{(y-2)^2} = 0$

$1\frac{1}{2}$

$\therefore$ The E+U Theo does not apply here.

Consider the autonomous DE $\frac{dy}{dx} = f(y)$, where $f(y)$ is the function whose graph is shown on the right.

SCORE: 0 / 6 PTS



[a] Find all equilibrium solutions of the DE and classify each as stable, unstable or semi-stable. **You must draw a phase portrait to get full credit.**

eq. sol'n: $-1, 3$

when $y < -1$, $\frac{dy}{dx} < 0$ $\quad y < 3$, $\frac{dy}{dx}$

$y > -1$, $\frac{dy}{dx} < 0$ $\quad y > 3$, $\frac{dy}{dx}$

[b] If $y = m(x)$ is a solution of the DE such that $m(4) = 2$, what is $\lim_{x \to \infty} m(x)$?

$$\lim_{x \to \infty} m(x) = \infty$$